[13.1] Let **G** be a set of elements with one element called “1”.

Part 1:

**Given** (1) 1*a* = *a* ∀*a*, (2) ∀*a* ∃ *a-1* such that *a-1 a* = 1, and

(3) Associative Law: *a* (*b c*) = (*a b*) *c* ∀ *a*, *b*, and *c.*

**Show** (A) *a* 1 = *a* ∀*a* and (B) *a* *a*-1 = 1 ∀*a.*

Part 2:

**Replace** (1) with (1’) : *a* 1 = *a* ∀*a*

**Show** this is not sufficient to imply (A) and (B)

**Solution Part 1:** From (2) ∃ 

(4)  So

(B) 

(A) 

**Solution Part 2:** Let **G** = {1, *x* } with multiplication defined by

12 = 1 *x* = 1 and *x*2 = *x* 1 = *x*.

(1’) : 

(2) : Define 

Then 

(3) : 



But (B) fails: 

Note: Part 2 solution is a simplification of deant’s simplification of Beckmann’s idea.